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Comments on Exclusive Electroproduction of Transversely Polarized Vector Mesons*

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Abstract

We discuss the electroproduction of light vector mesons from transversely polarized photons. Here QCD factorization cannot be applied as shown explicitly in a leading order calculation of corresponding Feynman diagrams. It is emphasized that present infrared singular contributions cannot be regularized through phenomenological meson distribution amplitudes with suppressed endpoint configurations. We point out that infrared divergencies arise also from integrals over skewed parton distributions of the nucleons.

In a phenomenological analysis of transverse vector meson production model dependent regularizations have to be applied. If this procedure preserves the analytic structure suggested by a leading order calculation of Feynman diagrams, one obtains contributions from nucleon parton distributions and their derivatives. In particular polarized gluons enter only through their derivative.

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In recent years exclusive electroproduction of mesons from nucleons has become a topic of broad interest. Experimental and theoretical advances have supported this development. At high energies a large amount of data has become available from experiments at CERN (NMC) and DESY (HERA, HERMES) (for references see e.g. [1, 2]). Further measurements are carried out at DESY, and discussed at CERN (COMPASS) [3] and TJNAF [4]. From the theoretical side a factorization theorem proven in ref.[5] let the basis for many investigations. It states that the underlying photon-parton sub-processes are dominated for longitudinally polarized photons and large photon virtualities, $Q^2 \gg \Lambda^2_{\rm QCD}$, by short distances and, hence, can be calculated perturbatively.

As already emphasized in [5], the interaction of transversely polarized photons cannot be treated in a framework based on QCD factorization. This is due to infrared sensitive contributions which e.g. result from large size quark-antiquark configurations in the produced meson. A straightforward QCD analysis of transverse vector meson production, as done for longitudinal ones (see refs.[8, 9, 10, 11, 12, 13, 14]), is therefore not possible. On the other hand, expected new data will provide more detailed information on these processes and help to investigate strong interaction dynamics at large distances.

Descriptions of exclusive meson production from transversely polarized photons rely on model assumptions which are needed to regularize the present, at least logarithmic, infrared singularities. In this note we outline several important properties of the involved production amplitudes: (i) different vector meson distribution amplitudes which enter in the production amplitude are related by Wandura-Wilczek type relations based on Lorentz invariance [6, 7]. These have significant implications for the nature of the existing infrared singularities. (ii) Infrared divergencies arise also from integrals over skewed parton distributions of nucleons. (ii) A model calculation of the vector meson production amplitude guided by Feynman diagrams yields an analytic structure which is richer than for meson production from longitudinal photons. In particular one obtains contributions which involve both parton distributions and their derivatives.

In the following we restrict ourselves to ρ production. An important ingredience of the corresponding amplitude are the meson distribution amplitudes. Following refs.[6, 7] we parametrize the meson-to-vacuum matrix elements of vector and axial vector [1] currents as:

$$\begin{aligned} \langle 0|\bar{q}(0)[0;x]\gamma_\mu q(x)|\rho(p,\lambda)\rangle = \; & p_\mu \quad \frac{e^{(\lambda)}\cdot x}{p\cdot x} f_\rho m_\rho \int_0^1 d\tau\, e^{-i\tau p\cdot x}\phi_{||}(\tau) \\ + \; & \left(e_\mu^{(\lambda)} - p_\mu \frac{e^{(\lambda)}\cdot x}{p\cdot x}\right) f_\rho m_\rho \int_0^1 d\tau\, e^{-i\tau p\cdot x} g^v(\tau)\,, \end{aligned} \tag{1}$$

and

$$\langle 0|\bar{q}(0)[0;x]\gamma_\mu\gamma_5 q(x)|\rho(p,\lambda)\rangle = \frac{1}{4}\varepsilon_{\mu\nu\rho\sigma} e^{(\lambda)\,\nu} p^\rho x^\sigma f_\rho m_\rho \int_0^1 d\tau\, e^{-i\tau p\cdot x} g^a(\tau)\,. \tag{2}$$

Here p^μ is the four-momentum of the ρ meson with invariant mass m_ρ and decay constant f_ρ. The vector meson polarization is specified by λ which corresponds to the polarization vector

[1] We use the convention of [15] for γ_5 and the epsilon tensor.

$e^{(\lambda)}$. The light-cone matrix elements, as well as the distribution amplitudes in eqs.(1,2), are defined at a certain renormalization scale μ which we suppress if convenient. Gauge invariance is guaranteed by the path-ordered exponential

$$[0; x] = \mathcal{P} \exp[-igx_\mu \int_0^1 A^\mu(x\eta)\, d\eta]$$

which reduces to 1 in axial gauge $n \cdot A = 0$ (g stands for the strong coupling constant and A^μ denotes the gluon field). In light-cone gauge the twist-2 distribution amplitude $\phi_{||}(\tau)$ can be related to the wave function of the minimal quark-antiquark Fock state in a longitudinally polarized meson [16, 17, 18]. The twist-2 distribution amplitude for a transversally polarized ρ is determined by the matrix element of a chiral-odd tensor quark operator and does not contribute to the process considered here. In the following we use also the antisymmetric distribution $\Phi_{||}(\tau)$ given by [6]:

$$\Phi_{||}(\tau) = \frac{1}{2}\left[\bar{\tau}\int_0^\tau du\, \frac{\phi_{||}(u)}{\bar{u}} - \tau \int_\tau^1 du\, \frac{\phi_{||}(u)}{u}\right], \tag{3}$$

with $\bar{\tau} = 1 - \tau$.

As explained in [6, 7] twist-2 and twist-3 string operators contribute to the distributions $g^v(\tau)$ and $g^a(\tau)$. Lorentz invariance leads to the presence of twist-2 contributions which are given by Wandzura-Wilczek type relations:

$$g^v(\tau) = \frac{1}{2}\left[\int_0^\tau du\, \frac{\phi_{||}(u)}{\bar{u}} + \int_\tau^1 du\, \frac{\phi_{||}(u)}{u}\right], \tag{4}$$

$$g^a(\tau) = 2\left[\bar{\tau}\int_0^\tau du\, \frac{\phi_{||}(u)}{\bar{u}} + \tau \int_\tau^1 du\, \frac{\phi_{||}(u)}{u}\right]. \tag{5}$$

Both, $g^v(\tau)$ and $g^a(\tau)$ are symmetric functions of τ. Twist-3 contributions are related to matrix elements of three-particle quark-gluon-quark operators [7] and will not be considered here.

In general one can assume $\sigma_\phi = \int_0^1 du\, \phi_{||}(u)/u$ is different from zero. This is quite natural since $\sigma_\phi = 0$ can be fulfilled, if at all, only at one particular scale μ due to the scale dependence of $\phi_{||}(u;\mu)$ [6]. As a consequence one finds from eqs.(4,5) in the limit $\tau \to 0$: $g^v(\tau) \sim \sigma_\phi$, $g^a(\tau) \sim \sigma_\phi\, \tau$ and $\Phi_{||}(\tau) \sim \sigma_\phi\, \tau$.

The amplitude $\mathcal{M}^{\gamma^*_\perp \to \rho_\perp}$ for ρ meson production from transversally polarized virtual photon can be split into parts,

$$\mathcal{M}^{\gamma^*_\perp \to \rho_\perp} = \mathcal{M}_G^{\gamma^*_\perp \to \rho_\perp} + \mathcal{M}_q^{\gamma^*_\perp \to \rho_\perp} + \mathcal{M}_{\Delta G}^{\gamma^*_\perp \to \rho_\perp} + \mathcal{M}_{\Delta q}^{\gamma^*_\perp \to \rho_\perp}, \tag{6}$$

involving unpolarized and polarized generalized quark and gluon distribution functions, respectively. For convenience we concentrate in the following on the gluon contributions. A straightforward calculation of leading order Feynman diagrams along the lines of ref.[12]

gives for the amplitude which involves the unpolarized gluon distribution:

$$
\begin{aligned}
\mathcal{M}_G^{\gamma_\perp^* \to \rho_\perp} &= i \frac{g^2}{32 N_C} \frac{f_\rho m_\rho}{\bar{Q}^2} \frac{\bar{N}(P', S') \hat{n} N(P, S)}{\bar{P} \cdot n} \; E \cdot e^* \Bigg\{ \\
& \mathcal{I}_1 \int_{-1}^{1} du \, G(u, \xi) \left[\frac{\bar{\omega}}{(\xi - u - i\epsilon)} + \frac{\bar{\omega}}{(\xi + u - i\epsilon)} \right] \\
&+ \; \mathcal{I}_2 \int_{-1}^{1} du \, G(u, \xi) \left[\frac{1}{(\xi - u - i\epsilon)^2} + \frac{1}{(\xi + u - i\epsilon)^2} \right] \Bigg\} , \qquad (7)
\end{aligned}
$$

while the contribution of the polarized generalized gluon distribution reads:

$$
\begin{aligned}
\mathcal{M}_{\Delta G}^{\gamma_\perp^* \to \rho_\perp} &= -\frac{g^2}{32 N_C} \frac{f_\rho m_\rho}{\bar{Q}^2} \frac{\bar{N}(P', S') \gamma_5 \hat{n} N(P, S)}{\bar{P} \cdot n} \; \varepsilon_{\mu\nu\alpha\beta} E^\mu e^{\nu\,*} \frac{n^\alpha n^{*\beta}}{n \cdot n^*} \\
&\times \; \mathcal{I}_2 \int_{-1}^{1} du \, \Delta G(u, \xi) \left[\frac{1}{(\xi - u - i\epsilon)^2} - \frac{1}{(\xi + u - i\epsilon)^2} \right] . \qquad (8)
\end{aligned}
$$

Here E^μ denotes the polarization vector of the virtual photon. $N(P, S)$ and $\bar{N}(P', S')$ are the Dirac spinors of the initial and scattered nucleon, respectively, with the corresponding four-momenta P, P' and spins S, S'. The average nucleon momentum is $\bar{P} = (P + P')/2$, and the momentum transfer is $r = P - P'$. The produced meson carries the four-momentum q' and $\bar{q} = (q + q')/2$. Furthermore, we have introduced the variables $\bar{Q}^2 = -\bar{q}^2$, $\bar{\omega} = 2\bar{q} \cdot \bar{P}/(-\bar{q}^2)$ and $\xi = 1/\bar{\omega}$. Finally, n is a light-like vector with $n \cdot a = a^+ = a^0 + a^3$ for any vector a, and $\hat{n} = \gamma_\mu n^\mu$. The ρ meson distribution amplitudes (1,2) determine the integrals $\mathcal{I}_{1/2}$ as given explicitly in eq.(10). $G(u, \xi)$ and $\Delta G(u, \xi)$ stand for the skewed unpolarized and polarized gluon distributions [19, 10]. In the forward limit, $\xi \to 0$, they reduce to the ordinary unpolarized and polarized gluon distributions of the nucleon:

$$
\begin{aligned}
& \lim_{\xi \to 0} G(u, \xi) = u g(u) \, , \\
& \lim_{\xi \to 0} \Delta G(u, \xi) = u \Delta g(u) \, . \qquad (9)
\end{aligned}
$$

For simplicity we have omitted in eqs.(7,8) so-called K-terms [19, 10]. Their contributions to the production amplitudes (7,8) can be obtained simply by replacing the skewed gluon distributions by K-distributions (including corresponding Dirac pre-factors) [12].

The integrals $\mathcal{I}_1$ and $\mathcal{I}_2$ contain the dependence of the production amplitudes on the meson distributions g^a, g^v and $\Phi_{||}$:

$$
\begin{aligned}
\mathcal{I}_1 &= \int_0^1 \frac{d\tau}{\tau} \left(4 g^v(\tau) - 2 \frac{\Phi_{||}(\tau)}{\tau} + \frac{g^a(\tau)}{2\tau\bar{\tau}} \right) , \\
\mathcal{I}_2 &= \int_0^1 \frac{d\tau}{\tau} \left(2 g^v(\tau) + \frac{g^a(\tau)}{2\tau\bar{\tau}} \right) . \qquad (10)
\end{aligned}
$$

Both, $\mathcal{I}_1$ and $\mathcal{I}_2$ are divergent due to the behavior of the integrands at $\tau \to 0$. These infrared divergences make QCD factorization impossible [5, 10]. Currently no QCD framework is available to deal with this problem of infrared sensitive contributions. As a consequence all predictions for ρ production from transversely polarized photons are model-dependent. One of the points we want to emphasize in this note is, that a modification of the end-point behavior of the the twist-2 amplitude $\phi_{||}(\tau)$ cannot cure the infrared divergence discussed above. This is a direct consequence of the Ball-Braun relation (4,5).

For the asymptotic distribution amplitude $\phi_{||}(\tau) = 6\tau\bar{\tau}$ the integrands in eq.(10) are proportional to $12/\tau$ and $9/\tau$, respectively. One, therefore, might expect that any phenomenological regularization which is applied to render the integrals finite, see eg. [20, 21], leads to a ratio $\mathcal{I}_1/\mathcal{I}_2$ close to one.

Another important point is the dependence of the production amplitudes on the unpolarized and polarized gluon distributions. According to eq.(7) the unpolarized skewed gluon distribution enters $\mathcal{M}_G^{\gamma_\perp^* \to \rho_\perp}$ through

$$\int_{-1}^{1} du\, G(u,\xi) \left[\frac{1}{(\xi - u - i\epsilon)} + \frac{1}{(\xi + u - i\epsilon)} \right], \tag{11}$$

and

$$\int_{-1}^{1} du\, G(u,\xi) \left[\frac{1}{(\xi - u - i\epsilon)^2} + \frac{1}{(\xi + u - i\epsilon)^2} \right]. \tag{12}$$

The first integral is also present in the leading-twist production amplitude of ρ mesons via longitudinally polarized photons (see e.g. [12]). The second contribution, which involves the square of $(\xi \pm u - i\epsilon)$ in the denominator, has not been considered before [20]. We believe that any model of transverse ρ production should include both contributions. Note that the integrals (11) and (12) are well defined only if $G(u,\xi)$ and its first derivative are continuous at $u = \xi$. Although little is known about properties of skewed parton distributions from first principles, at least the asymptotic distribution $G(u,\xi;\mu \to \infty)$ fulfills this requirements [22].

The polarized gluon distribution enters the production amplitude (8) via the integral

$$\int_{-1}^{1} du\, \Delta G(u,\xi) \left[\frac{1}{(\xi - u - i\epsilon)^2} - \frac{1}{(\xi + u - i\epsilon)^2} \right]. \tag{13}$$

Also this integral exists only if $\Delta G(u,\xi)$ and its derivative are continuous at $u = \xi$ which, again, is suggested by the asymptotic solutions of the corresponding QCD evolution equations.

The dependence on the skewed gluon distribution in eq.(13) is identical to the one found in J/Ψ production [23], but at variance with the model proposed in [21]. Integrating eq.(13) by parts shows that $\mathcal{M}_{\Delta G}^{\gamma_\perp^* \to \rho_\perp}$ depends on the derivative of $\Delta G(u,\xi)$ rather then on $\Delta G(u,\xi)$ itself. The imaginary part of $\mathcal{M}_{\Delta G}^{\gamma_\perp^* \to \rho_\perp}$ is, for example, proportional to

$$\mathrm{Im} \int_{-1}^{1} du\, \Delta G(u,\xi) \left[\frac{1}{(\xi - u - i\epsilon)^2} - \frac{1}{(\xi + u - i\epsilon)^2} \right] = -2\pi \frac{\partial}{\partial u} \Delta G(u,\xi)|_{u=\xi}. \tag{14}$$

It can be argued that $\Delta G(u,\xi)$ is proportional to $u\Delta g(u)$ for small ξ and $u \gtrsim \xi$ [12, 24]. The magnitude of the imaginary part (14) in the small-ξ region depends then crucially on the small-u behavior of $\Delta g(u)$. Due to the derivative in eq.(14) the contribution of polarized gluons to ρ production is small if $\Delta g(u)$ has a a strong singularity for small u as suggested e.g. in [25]. To the contrary, the contribution of polarized gluons can be large only if $\Delta g(u)$ does not rise fast at small u. This observation disagrees with the model calculation in ref.[21].

So far we have considered only the gluonic part of the production amplitude (6). The quark amplitudes $\mathcal{M}_q^{\gamma^*_\perp \to \rho_\perp}$ and $\mathcal{M}_{\Delta q}^{\gamma^*_\perp \to \rho_\perp}$ have a form similar to the gluon ones, but involve skewed unpolarized and polarized quark distributions $F(u,\xi)$ and $\Delta F(u,\xi)$. Both, unpolarized and polarized quark distributions enter through integrals involving denominators $(\xi \pm u - i\epsilon)$ and their square as in (11,12,13). However, for the asymptotic flavor singlet quark distribution [22]

$$F(u,\xi,\mu^2 \to \infty) = \frac{15}{2}\,\frac{N_F}{4C_F + N_F}\,\frac{1}{\xi^2}\,\frac{u}{\xi}\left(1 - \left(\frac{u}{\xi}\right)^2\right)\int_0^1 d\omega\,\left(\omega F_0(\omega,\xi) + G_0(\omega,\xi)\right)\,, \quad (15)$$

one finds an important difference as compared to the gluon case: the real part of the integral

$$\int_{-1}^{1} du\, F(u,\xi)\left[\frac{1}{(\xi - u - i\epsilon)^2} - \frac{1}{(\xi + u - i\epsilon)^2}\right]\,, \quad (16)$$

which is present in $\mathcal{M}_q^{\gamma^*_\perp \to \rho_\perp}$, is infrared singular. It seems plausible that this divergence occurs for any normalization scale μ [2]. This observation makes clear that a phenomenological regularization of the integrals which involve meson distribution amplitudes does not necessarily result in a finite result for the complete production amplitude.

In summary, we have shown via an explicit calculation of leading order Feynman diagrams how infrared singular contributions enter in the production of light vector mesons through the scattering of transversely polarized photons. They arise through integrals over involved meson production amplitudes and skewed parton distributions. The former cannot be regularized through phenomenological meson distribution amplitudes with suppressed endpoint configurations. This is a direct consequence of Lorentz invariance which provides relations between different vector meson distributions.

In a phenomenological analysis of data on transverse ρ meson production model dependent regularizations have to be applied. If this procedure preserves the analytic structure suggested by a leading order calculation of Feynman diagrams, one obtains amplitudes which contain contributions from parton distribution functions and their derivatives. In particular polarized gluons enter only through their derivative.

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